Modern Galileos

Let us now try to do some measurements on an accelerating object. This activity is inspired by a landmark experiment done by the famous scientist Galileo in the 17th century. All you need to do is to arrange a plank, a ball to roll on it, a scale to measure distance and a stopwatch to measure time intervals. You can use the back of a blackboard, table-tops or classroom desks as planks for this activity. Make sure that the plank is more or less flat, without dents and is at least 1 meter long (it will be difficult to take the readings if the plank is any shorter). Divide the plank into two equal segments of 45 cm by drawing lines across its width. Also mark a small line 3-4 cm before the first line. This will be the mark for releasing the ball (Fig. 41). Now, keep the plank on the floor or a flat table and raise one end by 3-4 cm. The plank will then be inclined at an angle of about two degrees to the horizontal. The accompanying pictures show one such arrangement with two segments marked on a plank. You can use small rubber or plastic balls, or even glass marbles for rolling down the inclined plane. The experiment consists of rolling

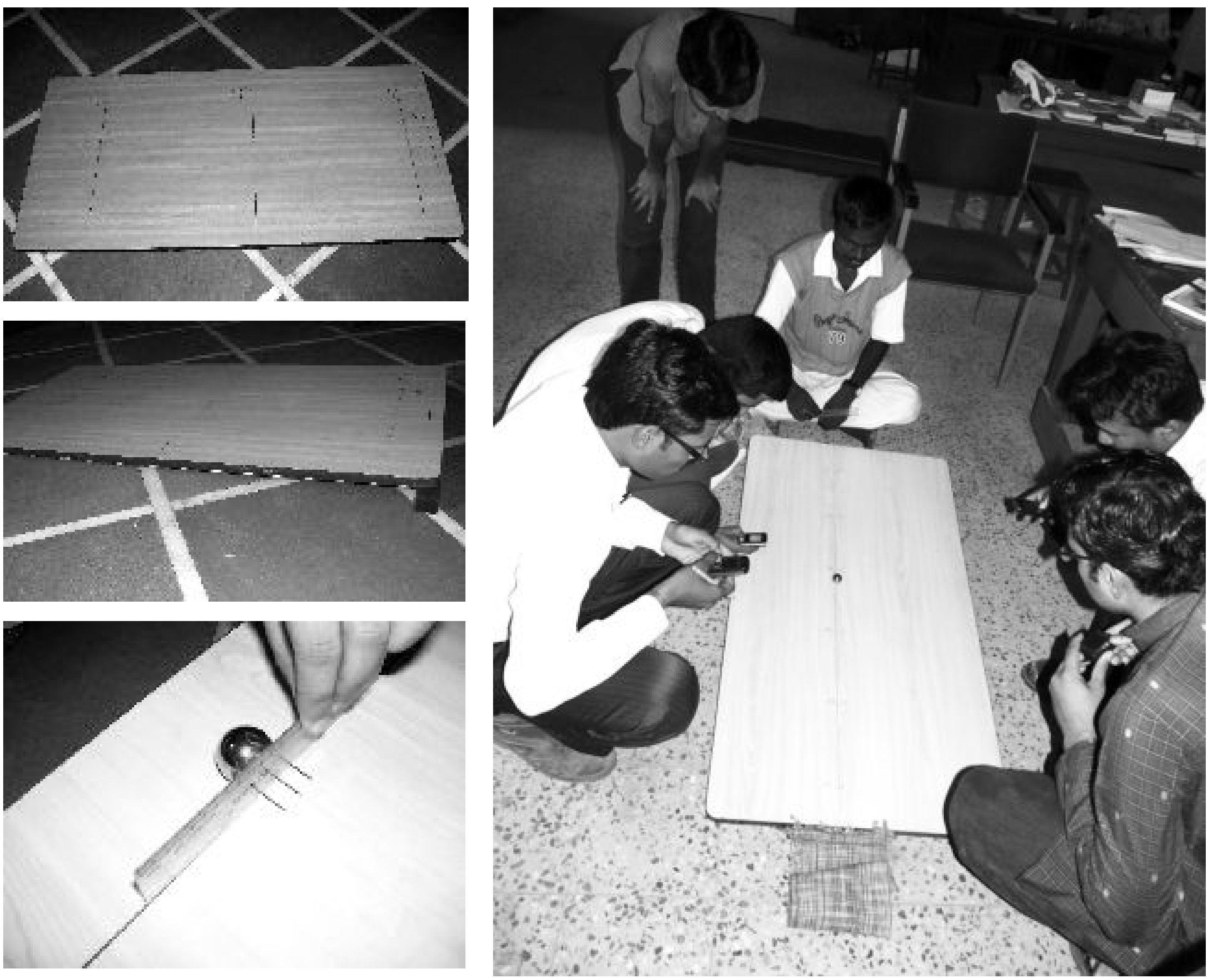


Fig. 41 A group of teachers doing the inclined plane activity. The picture on the bottom left shows a ruler to keep the ball at the starting point. When the ruler is removed, the ball starts rolling from this fixed starting place and with minimum starting speed.

the ball down the inclined plank and measuring the time it takes to cross each segment. For this, assign one person with a stopwatch to each segment. A third person starts the ball rolling by releasing it at the starting line. The time taken for the ball to cross each segment is noted down in a table similar to the one given below (Table 11). Once the time to cross each segment has been noted down, the average speed in each segment can be calculated by dividing the segment length by the time taken.

Like in any experiment, some precautions need to be taken to get reliable data. Firstly, the starting point should be the same in each trial. For example, in the bottom left picture of Fig. 41, three small lines can be seen drawn perpendicular to the starting line. These are to ensure that the ball is always released from the same place. The starting point should be selected by rolling the ball down the plane from various points along the starting line, and selecting the spot from where the ball rolls straight down the plank and does not veer to the sides. Ensuring that the starting point is the same takes care of any difference between trials arising due to warps, dents or scratches on the plank. In this way we try to minimise systematic errors. For more details on systematic errors and the ways in which they can be avoided, refer to appendix 3.

Secondly, care should be taken that the ball is not given an initial push when being released, that is, it must be released from a stationary position. This can be done with some practice. Using a ruler to hold the ball in place and releasing the ball by lifting the ruler off the board also works well. Thirdly, the experiment should be repeated several times. This is to get an estimate of the random error.

Table 11

Segment Length: _____ cm

Material of the ball: ___________

Tilt given to the plank: _____ degrees

Segment no.	Time taken to cross the segment(s)	Average speed in the segment (m/s)
First		
Second		

Alternatively, the readings can also be taken as follows: one student notes the time taken for the ball to cross the first segment, and another notes the time taken for the ball to cross two segments. The time taken to cross each segment can then be calculated from the two values.

You can divide the plank into three segments and repeat the experiment. You can do this if the plank is long enough and the students become skillful in using the stopwatch to get accurate values for time.

Table 12

Segment no.	Time taken to cross the segment(s)	Average speed in the segment (m/s)
1	2.72	0.18
2	1.89	0.25
3	1.51	0.32
4	1.33	0.36
5	1.15	0.42
6	0.93	0.52

Table 12 shows the data from such an experiment done with a much longer aluminium plank during a training workshop. The plank was divided into six segments of 48 cm each.

Analysis:

1. Look at the second column of Table 12. The time taken to cross each segment decreases continuously as the ball progresses down the plank. Since the segments are all of the same length, this means the ball moves faster in the later segments. Based on the definitions of speed and acceleration we arrived at earlier, we deduce that the ball is undergoing accelerated motion and that this acceleration is positive.

2. The average speeds calculated in the third column of Table 12 show that the average speed increases as the ball moves down. Now, we can imagine what would have happened if the plank had been divided into more segments. The same length could have been divided into 9 segments. Then too we would have found the average speed for each segment to be higher than that for the preceding segment. Subdividing the segments like this (in our imagination), we can

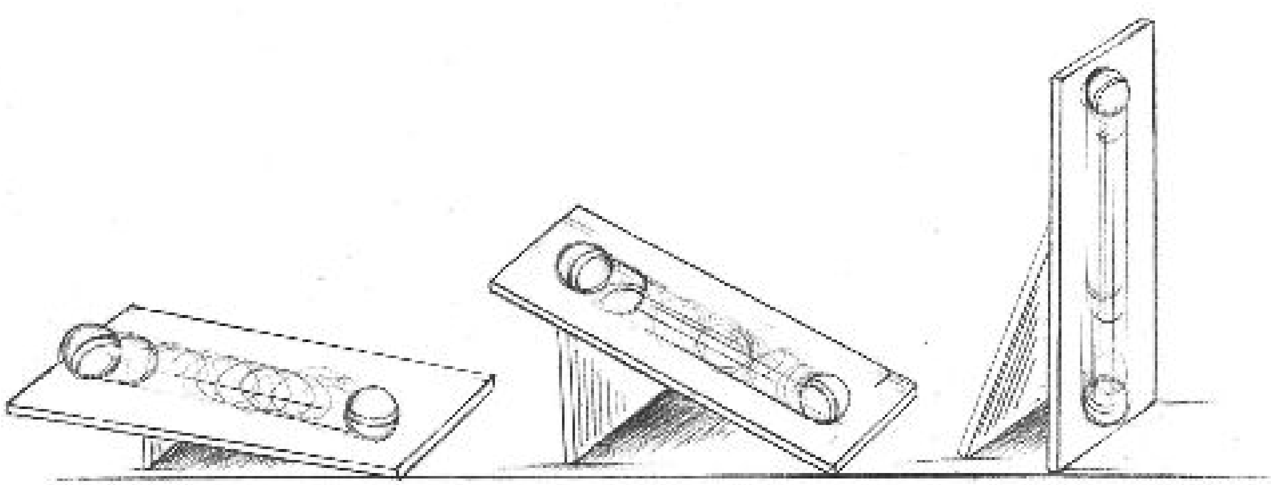


Fig. 42 Ball rolling down the planks inclined at various angles, including the vertical.

argue that the instantaneous speed (i.e. the average speed measured over the smallest time interval) increases continuously as the ball rolls down the plank.

3. Change the angle of the plank and repeat the experiment (Fig. 42). You will find that although the times to cross the segments change, the time taken by the ball to cross the second segment is always less than the time taken to cross the first. This will hold true even if the plank is held vertical, a situation equivalent to dropping a ball in air, that is, free fall. Thus, we can conclude that free fall is also accelerated motion.

Imagine trying to measure the acceleration of, say, an ant, or someone taking a walk, a rolling ball or a stone thrown in the air. Discuss amongst yourselves the best way to do this.

Acceleration Around Us

Which do you think has a higher speed—a giant wheel in a *mela* or a Shatabdi Express train? If you guessed the Shatabdi, then you are right. The average speed of a Shatabdi Express train is nearly 10 times that of a typical giant wheel. Yet, the thrill you get on a giant wheel is absent while seated in the air-conditioned comfort of this train. One reason may be that a person sitting in a uniformly moving train feels almost no acceleration, whereas in a giant wheel the acceleration can be as much as 1.5 times that felt by a freely falling stone. But even in the train, we can feel the change in motion when the train is speeding up, slowing down or turning along a curve. This change in motion can be felt more starkly when traveling in a bus; when the driver brakes suddenly, the jerk we feel is the (negative) acceleration of the bus coming to a stop.

In a giant wheel, we experience forces in different directions when we are at the top, the middle and the bottom of the ride. Along with this, the sensation of moving up and down adds to the thrill. We will learn more about the action of force on motion in part 2 of this series of modules.

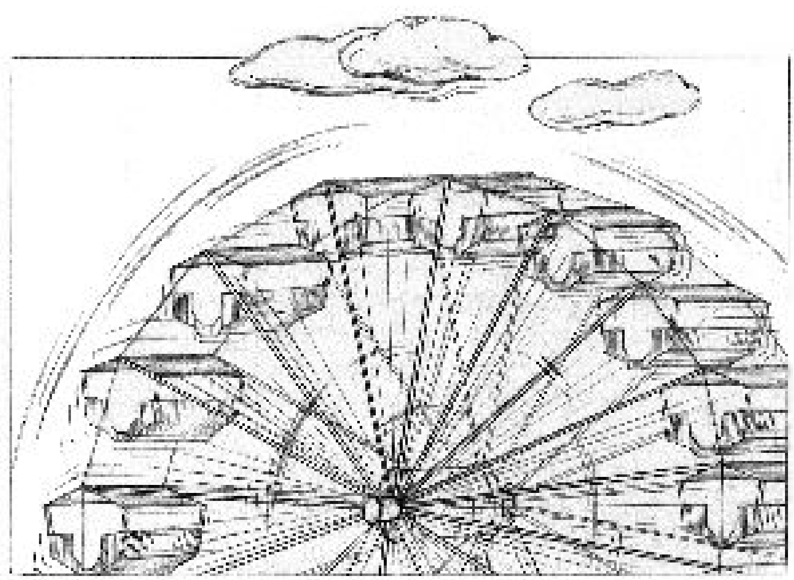


Fig. 43 Various positions in a giant wheel

Some typical values for acceleration are shown in Table 13:

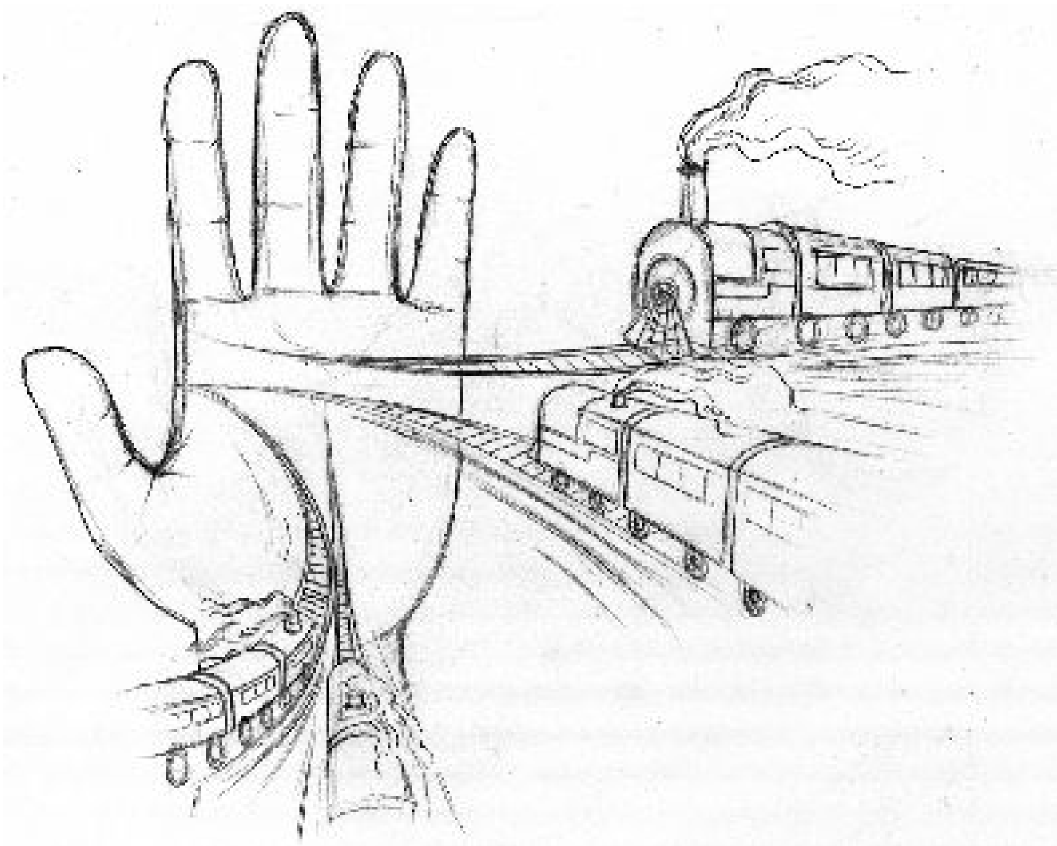
Table 13

Sl. No.	Motion	Typical acceleration (m/s^2)
1	Falling stone (free fall, on Earth)	9.8
2	Falling stone (on the Moon)	1.6
3	Lift in a shopping mall	1
4	Bullet shot out of a gun	1,00,000
5	Pick-up of a family car	3
6	Pick-up of a racing car	170

Is There an Accelerometer?

You might be wondering if, like the speedometer, vehicles also have an instrument that measures and displays acceleration. The answer is 'no', there is no such instrument on normal vehicles. The reasons probably are, (a) in the normal course of driving, information about speed is sufficient for the driver, and (b) as you have seen, it is much more complicated to measure acceleration. However, there are instruments called accelerometers available and you will see them if you visit a car factory. They are used in testing the performance of engines as well as brakes. Accelerometers are also used to measure vibrations in cars, machines, buildings, process control systems and safety installations. Specifically configured accelerometers called gravimeters are used to measure changes in gravity. Accelerometers are now being used to track the movements of animals, in sports training, rockets and video games. They can also be seen in the latest mobile phones.

Predicting Motion



Until now we have described motion in two ways: one, by giving values of the measured position or speed at different times, and the other showing the same information graphically. So far, so good. But we have to remember that one aim of science is also to predict how any process is going to evolve with time, if the initial conditions are known. Coming to the question of where we would need to use the calculations of motion, just think of our vast railway network. How would we coordinate the running of trains on the same track if we could not calculate the position of a moving train at different times? Can you think of some more examples?

By this time the students would have got some familiarity in quantifying the motion of phenomena which they observe all around them. Now, they can be introduced to the correspondence of a physical process to an appropriate mathematical equation, another powerful tool used by science. These equations are, for the moment, described in the scalar form for one-dimensional motion.

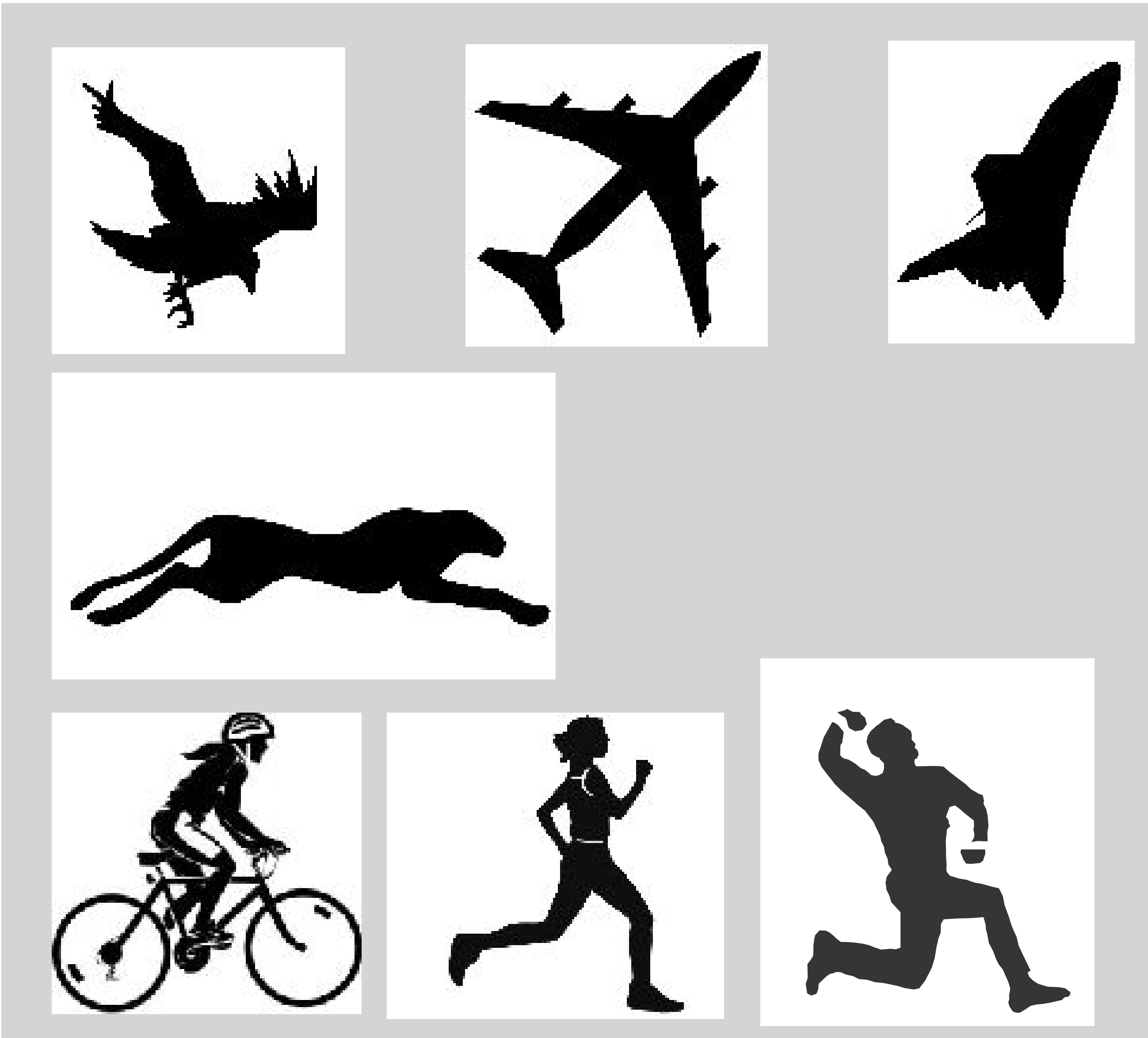


Fig. 44 Various moving objects

To take a specific case, a bus is traveling in a certain direction at a uniform speed of 40 km/h. Can we tell how far the bus will have traveled in six and a half hours if it kept to the same speed? Or, a bus is moving at a speed of 20 km/h and is given an acceleration of 120 km/h^2 in the same direction. Can we predict what its speed will be after 30 minutes? Conversely, can we find out how much time will it take to increase the speed of the bus to 60 km/h?

To answer such questions, science uses another tool, which is the mathematical representation of objects, properties and physical processes. We have already seen that we can define and measure some properties of linear motion, namely distance, time, speed and acceleration. Now we shall see how to mathematically define the relations between these quantities. Such relations are nothing but equations connecting the various mathematical quantities.

In describing acceleration earlier on page 49, we had written the following equation:

$$a = (v-u) \div t$$

where ‘u’ is the initial speed, ‘v’ the final speed, ‘t’ the time taken to change the speed and ‘a’ the acceleration. These terms can be rearranged to get the more popular form:

$\boxed{v = u + at}$ which is also known as the **first equation of motion**.

If an object has **constant acceleration**, then we know from the above discussion that its speed changes linearly with time (Fig. 40 on page 48). In this case, the average speed is equal to (u + v) / 2. A general proof of this is beyond the scope of this module, but do remember this averaging works **only if acceleration is constant**. We know from our prior discussion that the average speed = distance covered/ time taken. If we denote distance by ‘s’, then we can write:

$$(u + v) \div 2 = s \div t$$

This can be rewritten as:

$$s = (u + v) \text{ X } t \div 2$$

Substituting for v from first equation of motion, we get:

$$s = (u+ u + at) \text{ X } t \div 2$$

$\boxed{s= ut+at^2 \div 2}$ which is also known as the **second equation of motion**.

We can also combine the above two equations to get a third relationship in terms of only ‘v’, ‘u’, ‘s’ and ‘a’.

From the first equation,

$$v= u + at$$

giving $\quad t = (v – u) \div a$

And from second equation of motion,

$$s = ut + ½ at^2$$

Substituting for ‘t’ in the second equation, we get:

$$s = u (v-u) \div a + ½ a (v-u)^2 \div a^2$$

Then the equation can be simplified (try to do this yourself) to the following, more common form:

$\boxed{v^2 = u^2 + 2 as}$ which is also known as the **third equation of motion**.

If the motion of an object is retarded, you will have to take '-a' as the magnitude of a.

At this point students should be made to do some problems using these equations. Standard textbook problems deal with the movements of balls, stones and vehicles which may seem a little artificial to them. To kindle their interest, two projects are given below. After working on the projects they can be asked to do some of the problems from the problem set.

For the projects, the students have to make a written schedule which can be displayed on the wall like a poster. They can make it attractive by putting in drawings and pictures as well. You can also make up more such examples and hold a competition between different groups. These projects will give them some practice in doing calculations using the equations of motion.

From here they can go on to do the more conventional problems given in appendix 5.

Project 1: Shakkarpara Express

Remember our vast railway network? The railway timetable is made by calculating how much time each train will take between two stations, how much time-gap there should be between two trains on the same track, etc. All these calculations use the very same relationships that we have been discussing. If you like, you can also try your hand at designing an imaginary rail network.

In the times of the Rajas, the different Indian kingdoms had each their own separate railway network. Let us imagine a small but rich *jagir* with three villages. The *jagirdar*, Seth Shakarkand, lives in Shakkarpara, grows sugarcane in Gannaganj, and the sugar mill is in the third village, Milleria. The *jagirdar* wants a railway to travel between the villages and transport sugarcane to the mill. He calls his engineer and tells him to make the train schedule for the sugarcane season:

1. Shakarkand's supervisor and some more employees have to go from Shakkarpara in the morning to reach Gannaganj by 8 a.m and Milleria before 9 a.m.
2. Shakkarkand wants to go around mid-day to Gannnaganj, spend two hours there, then go to Milleria, spend two hours there and then come back.
3. Loading sugarcane onto the train takes two hours. Unloading at the mill takes one hour. If possible, the train should make two trips every day between Gannaganj and Milleria so that sugarcane from other nearby farms can also be taken to the mill from Gannaganj.
4. In the evening, Shakkarkand's employees have to come back from Milleria and Gannaganj to Shakkarpara.

The small meter-gauge train has an average speed of 20 km/h. Gannaganj and Milleria are 20 km and 30 km west of Shakkarpara, respectively. Try to make a train schedule with a minimum number of trips.

Project 2: Picnic Pickup

Your group of five friends decides to go to Patalpaani (a nearby waterfall) for a picnic. The jeep driver has to be told when to pick you all up from your houses. You decide to make a schedule so that you can give the exact time to everyone. In residential areas, the jeep can only travel at an average speed of 20 km/h, but once it is on the main road, it can travel at 60 km/h. You want to spend five hours at the waterfall and your mother wants you back home by 6 p.m.

The distances are as follows:

Your house to Akash's house: 2 km

Akash's house to Priya's house: 1 km

Priya's house to Bholu's house: 1.5 km

Bholu's house to Ganga's place: 3 km

Ganga's place to the main road: 2 km

Distance from Ganga's place to the waterfall: 50 km

At each house, the jeep stops for five minutes for people to board, and during the return journey, it again stops for five minutes for them to get off. You can calculate the times needed for the jeep to go from one point to another. You can then find out the total time required for the trip, and accordingly tell the driver when to pick you up from your house. Fill in the schedule given below:

Driver reporting at your place: ....am

Jeep reaches Akash's house: ....am

Jeep reaches Priya's house: ....am

Jeep reaches Bholu's house: ....am

Jeep reaches Ganga's house: ....am

Jeep reaches waterfall: ....am

Jeep starts from waterfall: ....pm

Jeep reaches Ganga's house: ....pm

Jeep reaches Bholu's house: ....pm

Jeep reaches Priya's house: ....pm

Jeep reaches Akash's house: ....pm

Jeep reaches your house: ....pm

So, where do you think these relationships between distance, time, speed and acceleration are used? Two interesting examples are described in the boxes that follow.

What Do Bats and Submarines Have in Common?

Bats are flying mammals, half the time hanging upside down from trees. Submarines are sophisticated underwater vehicles which can stay submerged for months. So what possibly can be common between them? They both calculate the motion of sound pulses to find out how far objects are from them. Bats have very poor eyesight, but you must have seen how they are able to zoom around even in the dark. This is because they use sound rather than light to 'see' things. Bats emit high-pitched sound pulses, and when these pulses hit any object like a wall, branch of a tree or an insect, they are reflected back to the bats. The bat brain can sense the time elapsed between the emitted pulse and the reflected (echo) pulse, and thereby estimate the distance of the object from itself.

Human brains and sense organs do not possess these abilities, but we have developed instruments like radar and sonar which do the same thing. Submarines use underwater sonar to 'see' what is around them in the water. Sound pulses are continuously emitted and the time for them to return after being reflected is measured (Fig. 47). Knowing the speed of sound in water, the distance of the object is estimated. Not only that, by continuously monitoring the time taken for the reflected sound to return to the source, we can find out if the object is stationary, coming towards the submarine or going away from it. Sonars are used in ships to locate submarines and fishes in deep seas, and to even study the seabed. Try to find out more examples in real life where calculations of speed, distance, time and acceleration are used.

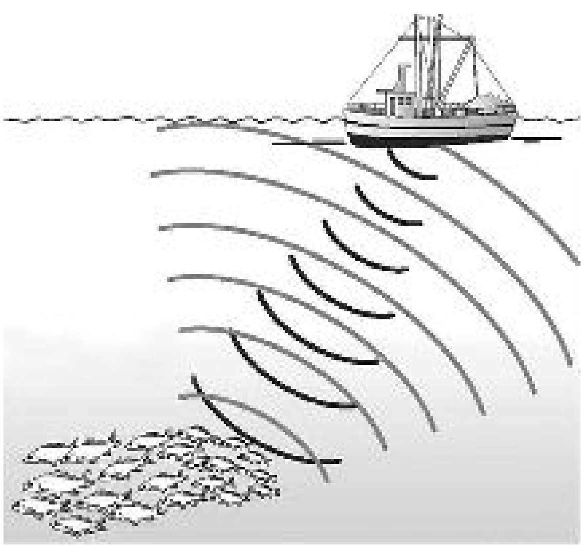


Fig. 47 Sound waves transmitted from the ship and reflected back from a shoal of fish

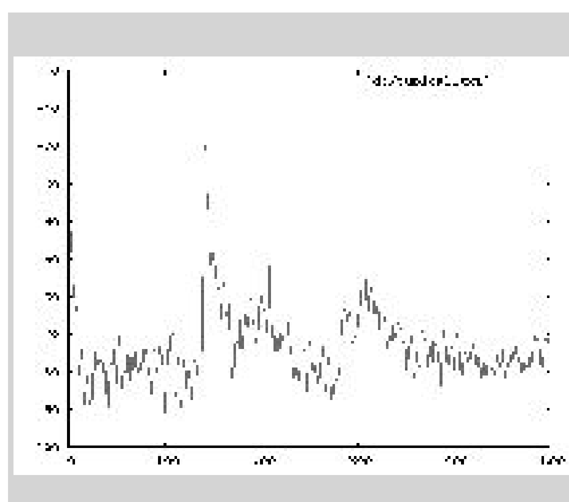


Fig. 48 Graph of active sonar data

How Far from Us is the Moon?

Have you ever wondered how the distance between the earth and its moon came to be known? Well, many techniques have been used throughout history, but the most accurate and direct measurement has been done by using the time taken for something to go to the moon and come back. Since the moon is very far from the earth, we would prefer the motion of this thing to be very fast so that the time taken to reach the moon and come back is not too great (scientists do try to be efficient and quick!). The fastest thing known to us is light. The astronauts who landed on the moon in 1969 placed huge mirrors called retroreflectors there (Fig. 46). Then light pulses from lasers on the earth were aimed at these mirrors. The time taken for the reflected light to return was determined (Fig. 45). Because the speed of light is known with a high degree of precision, the distance from the earth to the moon can be calculated using this simple equation:

Distance = (Speed of light × Time taken for light pulse to come back after reflection) ÷ 2

The time for a light pulse to go from the earth to the moon and back was around 2.5 s. Of course, the light pulse itself is of a much shorter duration than this (can you work out why this has to be so?). The average distance between Earth and its moon has thus been measured to be about 3,84,467 kilometers (238,897 miles). The earth-moon distance has been averaged because it varies a little over time—you might have read about the 'supermoon' occurring periodically when the moon is very close to the earth.

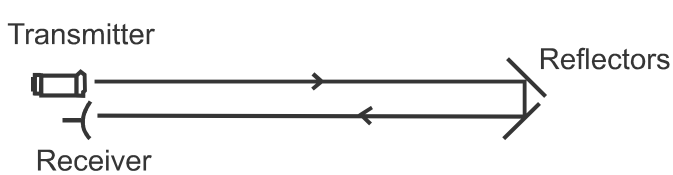


Fig. 45 Schematic diagram of a laser rangefinder

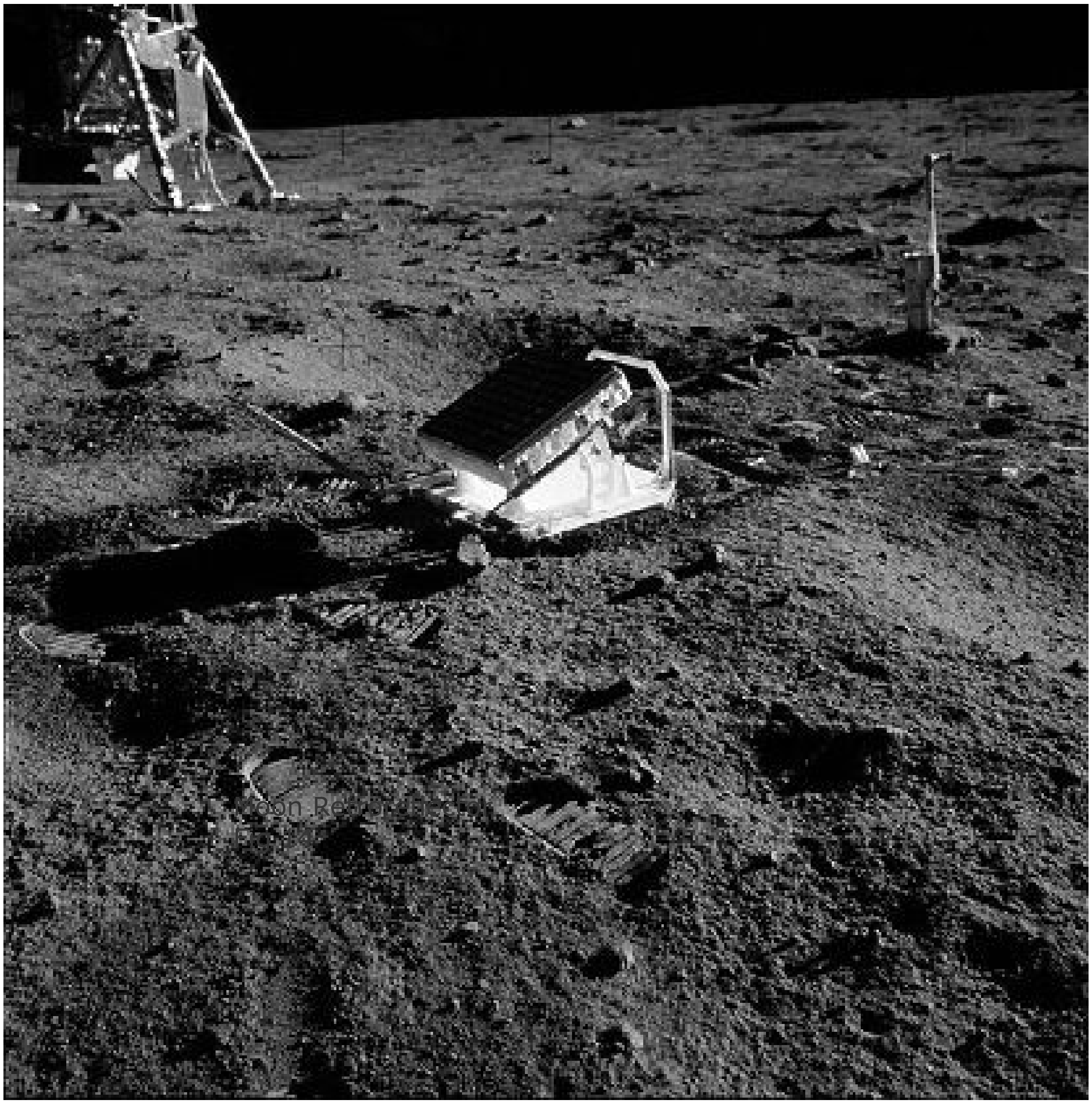


Fig. 46 Retroreflector placed on the moon by Apollo11 astronauts

By this point the students would have hopefully become interested enough in motion to be willing to extend themselves and go deeper into the subject. In the next module, we start by discussing what causes motion and what causes acceleration. We will also discuss the vector nature of velocity, acceleration and force, although the students might not yet have encountered vectors. In additon, the next module includes the historical development of the concepts of force and motion. This will help students in getting a conceptually correct understanding of the subject as well as serve as an example of how scientific theories develop.

Appendix 1

Science and the Scientific Method

This appendix is addressed to teachers. The evolution of the concepts of motion and force will be discussed in part 2 of this series where readers will get a flavour of the scientific approach. Two project ideas are given at the end of this appendix for teachers to use.

Have you ever wondered what science is and why is it considered different from other fields of study? One way of defining science is that it is a process of studying something scientifically or, in other words, that uses the scientific method. Thus, we have mathematical science, physical science, biological science and even social science. The common factor amongst all these 'sciences' is the scientific method.

The evolution of our understanding of motion and force is a very good example to see how science and the scientific method evolved over time. In the early civilizations, people used only their unaided senses to make observations and the philosopher-scientists of that time proposed a theory of motion based on such observations. One influential philosopher was Aristotle (384-322 BC). He claimed that the speed of a falling object increases in proportion to its mass (heavier objects fall faster than lighter ones). The logic of this claim was questioned by other philosophers, but a general belief in Aristotle's theory continued. Meanwhile, with progress in the techniques to measure time and distance, it was possible to carry out experimental tests of the theory of motion. Finally, in the 17th century, Galileo (1564-1642) showed by an experimental test and logical extrapolation that all objects dropped from the same height will reach the ground at the same time if there is no friction due to air. Several scientists continued to add their refinements to these findings with Newton (1643-1727) expressing the then available knowledge in the form of equations and laws. Newton proposed that the acceleration (rate of change in speed) of an object is proportional to the force applied on it. The mass of the object is an inherent property of the object and is constant. This became the basis for Newtonian mechanics which was considered correct and was expected to explain all the observed motions on Earth as well as all astronomical motions. However, over the course of time, some observations were found that did not fall in line with the predictions of Newton's laws. In particular, the observation that the speed of light is a constant (and does not depend on the frame of reference) called for a more comprehensive theory. Einstein, Lorentz and Poincare's works led to a more accurate theory, namely that of special relativity.

The theory that is currently accepted was proposed by Einstein (1879-1955). According to this theory, at very high speeds (near the speed of light, which is a constant), the mass of an object increases. This effect is noticeable only at very high speeds which we do not normally encounter. If you look at the dates in the preceding paragraph, you will see that from Aristotle to Galileo was a journey of 2000 years, Galileo to Newton about 85 years, and from Newton to Einstein more than 200 years! Over this time period, our understanding of motion was continuosly evolving. The initial understanding was that heavier objects fall faster and that this is their intrinsic nature. This was later modified, and the understanding was then that the action of a constant gravitational field, with no other forces interfering in it, will lead to all objects falling with the same acceleration. Galileo took these laws to apply on Earth and Newton's laws of gravity extended our understanding to the motion of heavenly bodies as well. But a further refinement was added by Einstein to limit these rules to bodies moving with speeds much lower than that of light.

This is not the end. If scientists find inconsistencies in this theory, then it will be reworked. However, until now no experimental measurements have contradicted its predictions. Scientists are continuously working on devising more accurate tests of the theory. It is possible that in the near future some experiment will reveal an error in the theory and then theorists will have to get down to working out a better theory which will also explain these new results. In any field of science, you will find a similar process of continuous evolution, of an understanding of any topic that is arrived at by the cumulative work of several people over a period of time. This may be termed the scientific method.

The scientific method currently rests on three foundation stones: (a) **accurate and objective observation,** (b) **mathematical and logical analysis** and (c) **modeling**. Depending on the problem to be solved, these three tools can be used in different ways and in different orders.

A sample recipe for the scientific method can be stated as follows:

Step No. 1: Ask a question—why, how, what, where, when, etc. about something. This question could be triggered by a phenomenon you observed directly or by thinking about something you might have read about or heard of.

Step No. 2: Find out whether anyone else has any information on the subject. You do this by asking people and by reading the available literature on the subject. This is called background research. If this exercise answers your question to 'your' satisfaction, you may look for another question. It is possible that later someone else comes up with a point that you had overlooked and the currently accepted answer is found to need some modification. This going back and forth is an integral part of science. A scientific theory (in contrast to dogma), is accepted as valid only until no new fact emerges to contradict it. The moment that happens, people start working towards a new theory.

Step No. 3: Construct a hypothesis to answer the question. That is, you make a guess, an intelligent one, based on current knowledge. The hypothesis could be just one, or you could construct multiple hypotheses that are several, alternative explanations for your observation.

Step No. 4: Devise a test or an experiment to check whether your hypothesis is true or false. Here, you have to consider several things. First, the test should be fair—it should be designed so that the results are not affected by any bias in the mind of the person performing the tests (this is actually hard to implement). Second, the test should be designed such that it does not give an ambiguous answer. This can most easily be ensured by designing the test so that its result is a number. That is, you actually measure something. This means that the hypothesis you constructed in step no. 3 should be such that a suitable test can be devised. This test is what we call an experiment.

Step No. 5: Analyse the results obtained in the experiment. This step has to be done very carefully and rigorously so that any effect of experimental bias or imperfection is taken care of. This step may give a clear answer. Or, it may show that the experiment needs improving, or that it is not possible to perform the tests with the available resources. In that case, one has to either redesign the experiment or construct another hypothesis. Steps 3-5 very often use the reductionist approach (discussed in the main text on page 15). Whether this approach has worked or not is finally decided in step 7.

Step No. 6: Communicate your results to as many people as possible. This is one of the most important steps without which no progress can be made in science. The reasons are twofold. One, it allows independent repetition of the experiment by others and testing of the hypothesis by different experiments. In other words, it allows the hypothesis to be debated by a world-wide community, thus increasing the probability of spotting any loopholes in it. The second reason is that once you have found out something, based on it, another person may be able to find something else. For this, scientists need to hone their communication skills.

Step No 7: Once the results of your experiment are clear and their analysis done, a model or theory about the phenomenon under consideration can be developed. This usually involves making a mathematical model of the phenomenon and can be based on the compiled results of several experiments. A theory is considered valid only if it can explain all previous observations in that field. Such theories can be useful in one or more of the following ways: (a) they are expected to explain a natural phenomenon and predict its evolution (e.g., prediction of the next earthquake), (b) they can help us find ways to control the phenomenon according to our requirements (e.g., preventing a comet from crashing into the earth), and (c) they can help us in developing new technologies to improve the quality of our lives (e.g., new sustainable sources of energy).

As you can understand, **the above recipe can have many variations depending on the type of study being done**. It may, (and usually will) require iterations at some step. All the steps need not be (and are mostly not) conducted by the same person or team, or even at the same place. Given below are some project ideas to give your students to do during their vacations. They must prepare a report giving a step by step description of how a problem was approached and solved. And if it was not solved, what the limitations were.

Projects:

1. What material is the best for making cooking utensils? (Hint: List all the factors affecting the choice of material like cooking method, cost, ease of cleaning, resistance to corrosion, evenness of heating, etc. Separate out the necessary requirements and the preferable qualities. Then search for materials which fulfill these requirements. The best choice may not be any one material. The outcome can be to specify which material is best for a given set of conditions. It can also be a blueprint for the properties which an ideal material should have. This project does not necessarily involve experiments, but the children can be encouraged to test some of the facts which they would have read about. For example, they can test whether it is possible to boil water in a paper cup, or whether water boils faster in an aluminium vessel as compared to one made of steel.)

2. My grandmother says that keeping peacock feathers (*morpankh*) in a room drives away house lizards. How would you go about testing this assertion (hypothesis)?

Appendix 2

Graphs

This appendix includes a more detailed discussion of the graphs used to describe motion. It is in two sections. Section A describes the fundamental principles of plotting graphs and can be used for students who have little or no familiarity with graphs. Section B discusses the graphical representation of motion in continuation with the material in the main module. Section A can be skipped if students are conversant with graphs.

A: Introduction to Graphs

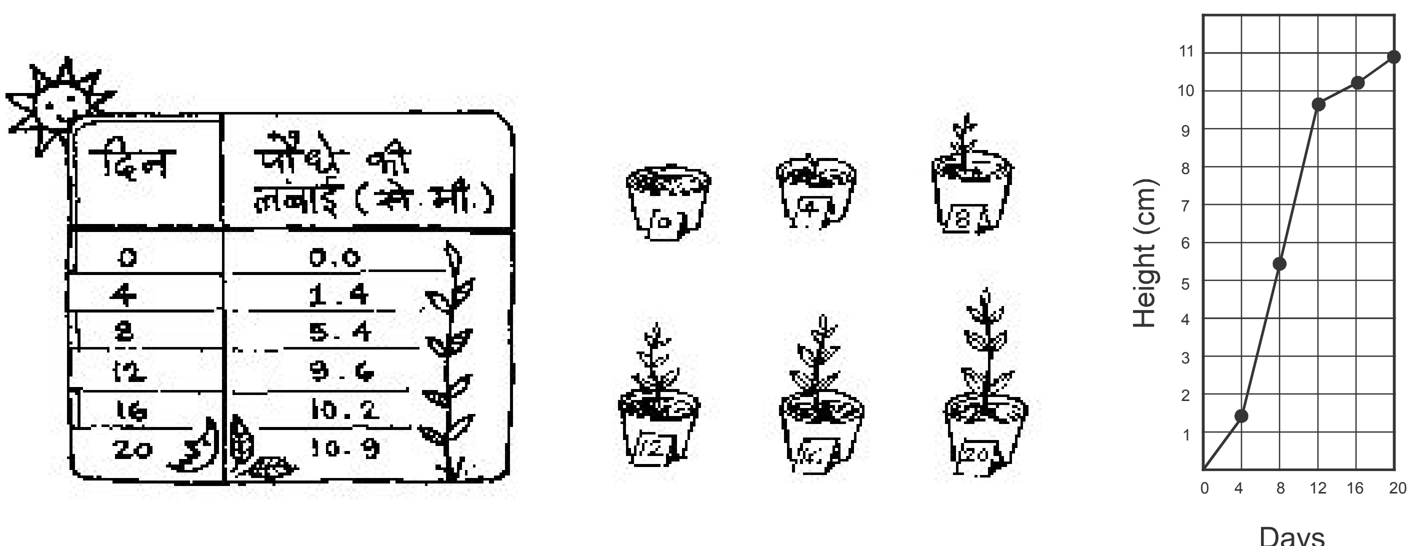


Fig. 1 Different ways of tracking plant growth

The growth of a plant (its height) is shown above in different ways: as a table, as a series of drawings and as a graph.

All three pictures tell us that over a period of 20 days the height of the plant increased continuously. However, the graph tells us so much more than the other two pictures. We can read off the measured height on every fourth day (same information as given in the table). At a glance, we can also see that the rate of increase in the plant's height has not remained constant over 20 days. The height of the plant increased at a slower rate after the 12th day. We can also estimate the plant's heights on days in-between. For example, on the 6th day the plant must have been around 3 cm in height, and on the 10th day around 7.5 cm.

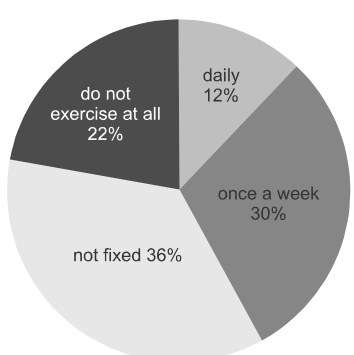
So what exactly is a graph and how do we make one? A graph is a pictorial representation of two variables, quantities that change according to some relationship between them (see the box below on types of graphs). In the example above, the number of days and the height of the plant were the two variables. The relationship between them is that the plant height increased as the days passed. In this example, ‘number of days’ is the independent variable, and ‘plant height’ the dependent variable as its value depends on how old the plant is. Making a graph is called ‘plotting’.

Types of Graphs

The graphs discussed so far are **line graphs** or line charts. They are used to depict the relationship between two quantities. This kind of graph was invented by the 17th century French philosopher-mathematician Rene Descartes (1596-1650) and provided the first systematic link between geometry and algebra. The x and y values of any point on such a graph are called its Cartesian coordinates, after Descartes. There are other kinds of graphs as well. **Bar graphs** can be used to compare more than two variables. For example, your school may use a bar graph to show the number of students passing in different classes every year. A **pie chart** is used to show the percentage distribution of values of some attribute in a given set of objects. You might have seen pie charts showing the results of opinion polls. Some examples are shown below.

Try to spot graphs in magazines, newspapers and on TV. Think about why a particular type of graph was used in that situation. Also, try to use graphs in reports you write for a project in any subject. You will find that it makes your reports better and easier to understand!

How often do you exercise?



Financial summary of a small scale company

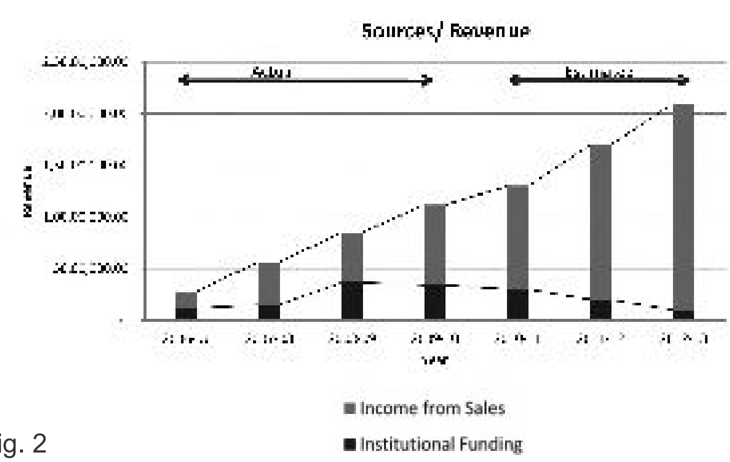


Fig. 2

Plotting a Graph

Let us go over the basic rules for plotting a line graph. Take the example of the relationship between the side of a square and its perimeter. Table 1 shows some data to be plotted. Start by taking a graph sheet and follow the instructions given below.

Table 1

No.	Length of the side of a square (cm)	Perimeter of the square (cm)
1	1	4
2	2	8
3	3	12
4	4	16
5	5	20

How to Draw the Axes and Plot the Data

1. Identify the two variables whose relationship is to be represented on the graph. In this case, the length of the side will be the independent variable and perimeter the dependent variable.

2. On the graph paper draw a horizontal line close to the bottom edge. Then draw a vertical line close to the left edge of paper such that it crosses the horizontal line at one point. The horizontal line is called the x-axis, and the vertical line the y-axis. Take care that both the lines are drawn on the dark lines of the graph paper (Fig. 3). The point where both these lines meet on the graph paper is called the 'Point of Origin'. The space below the x-axis and to the left of the y-axis is used for writing the description of the axes (see Fig. 3).

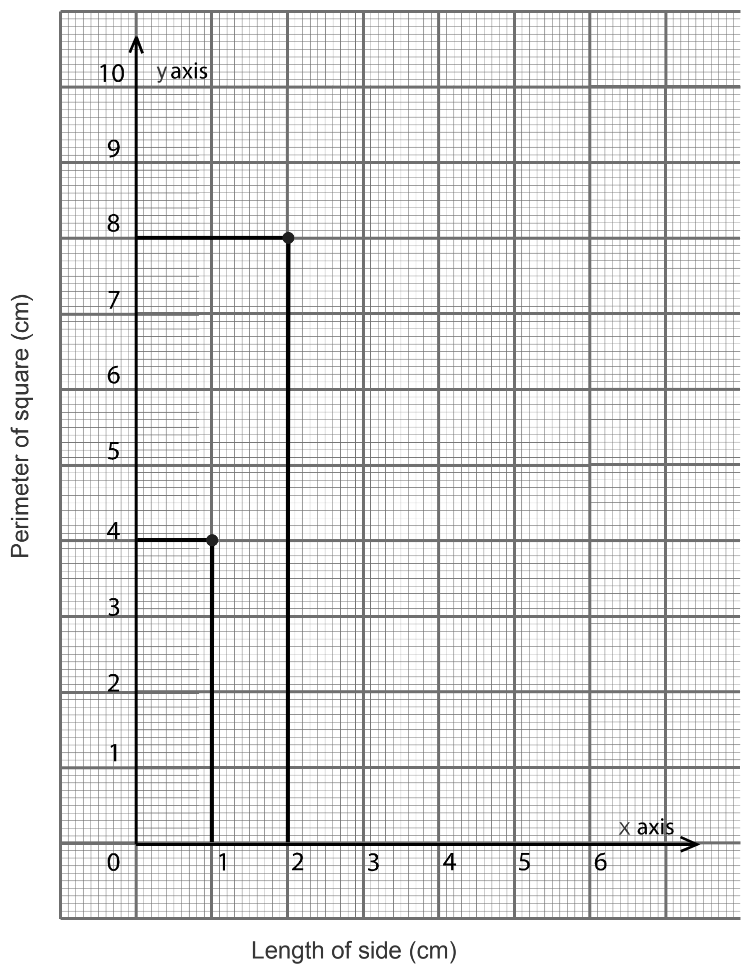


Fig. 3 Perimeter of a square vs. Length of its side

3. The independent variable (in this case, the length of the side of a square) is marked on the x-axis. The dependent variable (the square's perimeter) is marked on the y-axis.

4. Mark the point of origin as '0'. Make markings on the x-axis at 1 cm intervals and number the marks 1, 2, 3, 4, 5 and so on, from left to right. Note that in all the graphs the markings have to be equally spaced.

5. You have to plot the perimeter of the square on the y-axis. Look at the values for perimeter in table 1. The largest square has a perimeter of 20 cm. So, divide the y-axis into twenty 1 cm divisions and number them 1 to 20 from bottom to top, starting 1 cm away from the origin.

Plotting Data Points

1. Table 1 shows that a square with a side of length 1 cm has a perimeter of 4 cm. Since the length of the side of the first square is 1 cm, draw a vertical line on the 1 cm mark of the x-axis. This line should be parallel to the y-axis (Fig. 3).

2. The perimeter of this square is 4 cm. So, draw a horizontal line at the 4 cm mark of the y-axis. This line should be parallel to the x-axis (Fig. 3).

3. Draw a circle around the point where these two lines intersect each other. This is your first data point (Fig. 3). Data points are those points on the graph paper which represent the data given in a table.

4. Plot other data points on the graph paper for the remaining four squares given in the table, in the same manner.

5. Join these points using a ruler to get a graph line. Why should you join these data points with a straight line? Think this over for a while before reading further.

Joining two data points with a straight line is known as 'linear approximation'. This means that we assume that for values of the x variable between these two points, the y variable changes linearly. How is this useful? Let us say we want to know the perimeter of a square with a side of 4.5 cm. It is not given in the table. Ordinarily, we would have to calculate the value.

However, it is straightforward to read it from the graph. Draw a vertical line at the 4.5 cm point of the x-axis. Name the point where this line meets the graph line as 'A '(Fig. 4). Draw a horizontal line parallel to the x-axis from the point A towards the y-axis. Where does this line intersect the y-axis? Read this value on the scale given on the y-axis. This is the perimeter of the square with a side of 4.5 cm. With some practice you will not need to draw the lines, you will be able to read the values by using the printed lines on the graph paper itself.

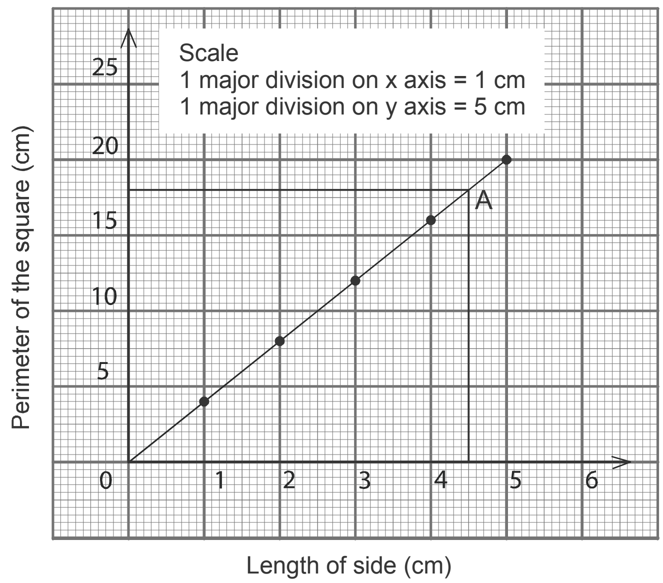


Fig. 4 Finding out the perimeter of a square with a given length from a graph

In this way, graphs are useful in giving us more information than the few values that are plotted using data given in a table. The process of estimating values between data points is called **interpolation**. Suppose we now want to know the perimeter of a square with a side of length 6 cm. Can this graph give us the answer? Extend the graph line with the help of a ruler. Now, find out the perimeter of a square that has a side of 6 cm by following the same procedure as was done for the square with side 4.5 cm. This process of extending the graph line to estimate the values beyond the known data is called **extrapolation**.

Choosing a Scale

The ease of getting information from graphs depends a lot on the scaling. Scaling means setting an appropriate unit of the independent variable as equal to one cm on the x-axis and similarly, setting an appropriate unit of the dependent variable equal to one cm on the y-axis. While setting the scale, keep the following in mind:

1. The scale is such that you are able to show the largest value on the graph paper.

2. The scale is such that almost all of the graph area is covered.

3. Choose easily divisible units so that reading between the marked points is easy.

We will illustrate these points using two data sets. The graph paper grid we will use is 13 cm by 7 cm in size. We mark the origin at a point 2 cm from the bottom and 1 cm from the left edge of the graph paper so the length of the x-axis is 6 cm and the length of the y-axis is 11 cm.

Example 1: Table 2 lists the areas of squares with sides of different lengths. The maximum value for the length of the side of a square (the independent variable) is 5 cm. So, for the x-axis we can take 1 major division on the graph to be equal to 1 cm of the length of the side of a square. The maximum value for area of a square is 25 sq. cm. How do we best fit all the values for area on the y-axis? If we take 1 major division on the graph as equal to 1 sq. cm area or 2 sq. cm area, some of the points will not fall within the graph sheet. So we can take 1 major division on the axis to be equal to 5 sq. cm area. This will give us a graph as shown in Fig. 5. You can see that some of the points fall in-between the thicker graph gridlines. Try some other scales and see which one is the most convenient to plot for this data set.

Table 2

No.	Length of one side of a square (cm)	Area of the square (sq. cm)
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25

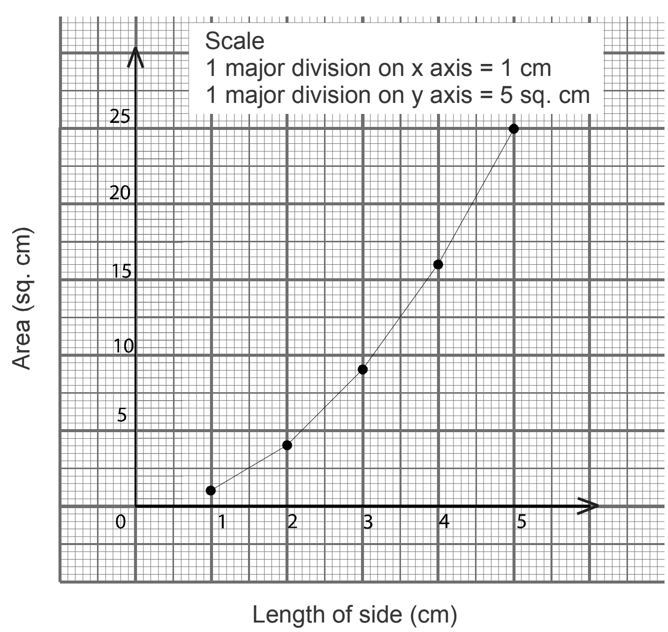


Fig. 5 Area of a square vs. Length of side

Example 2: The second example is that of wheat yield per acre. Here, the scales chosen are 1 major division on the x-axis = 2 acres and 1 major division on the y-axis = 2 tons. So you see, any kind of measurable quantity can be plotted on a graph.

Table 3

Wheat yield per acre		
No.	Size of field (in acres)	Wheat yield (in tons)
1	1	1.5
2	2	3
3	3	4.5
4	4	6
5	5	7.5
6	6	9
7	7	10.5
8	8	12
9	9	13.5
10	10	15

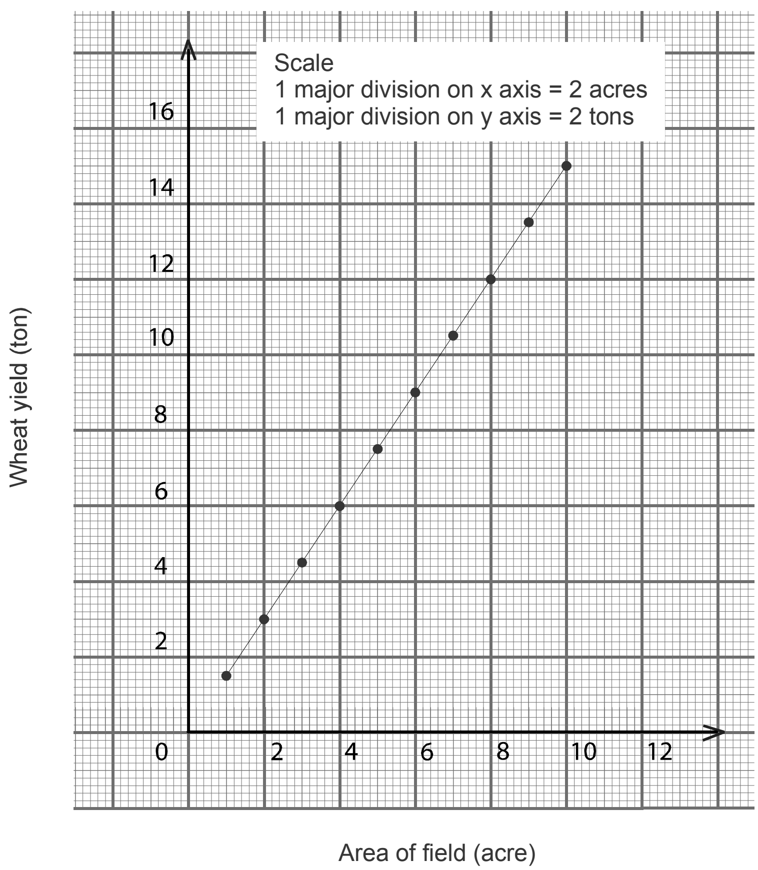


Fig. 6 Wheat yield for a given field area

Limitations of Graphs

Like any other mathematical tool, graphs also have limitations and must be used knowing what these are:

1. The only actual information in a graph is the plotted data points.
2. There is a limit to the precision with which the points can be plotted on and read from the graph.The least count of the measuring instrument you have used will decide the minimum spacing between two consecutive data points. Since you don't have the instruments to take data for the points in-between these two, you have to make a guess for these points. For the problems considered here, it is usually safe to assume that the graph between any two data points is a straight line joining them.
3. In case of extrapolated values, one should take care to check whether the estimated value is physically possible.

In the context of motion, in one glance a graph provides a visual representation of motion. Research has revealed that students' common misconceptions in interpreting graphs of motion arise from the fact that such graphs are confused with maps of the physical routes taken by the persons or objects in motion. They often visualise the end point of a graph line as the end of the trip, or the dead end of a road. The inclined line is thought to be the slope of the road, and the points of change in the slope are perceived as the points where a traveller turns. This issue is already addressed in the example of Ritu's walk discussed in the main text. You can also use the following example to test whether the children have understood the difference between a graph and a map, and whether they can read data correctly from a graph. These basics are essential before going on to more detailed discussions on the use of graphs in understanding motion. We suggest that some time be spent to discuss in detail the ideas that students have and to correct any misconceptions before proceeding further.

Let us take the example of Munni walking from her home to school. On the next page, the drawing on the right (not to scale) is a map of the route Munni takes. On the left is a distance-time graph of her journey marked with measurements made every two minutes. The actual measurements are shown by dots. Consecutive dots are joined by straight lines. From the two pictures, answer the following:

1. Can you estimate how long Munni takes to reach her school by looking at the map?
2. By looking at the graph, can you guess how many turns there are along the road from Munni's home to her school, or the point where the road crosses the river?
3. How much distance did Munni cover from the 8th minute to the 10th minute of her journey?
4. Did Munni cover equal distances in each two-minute interval of her journey?