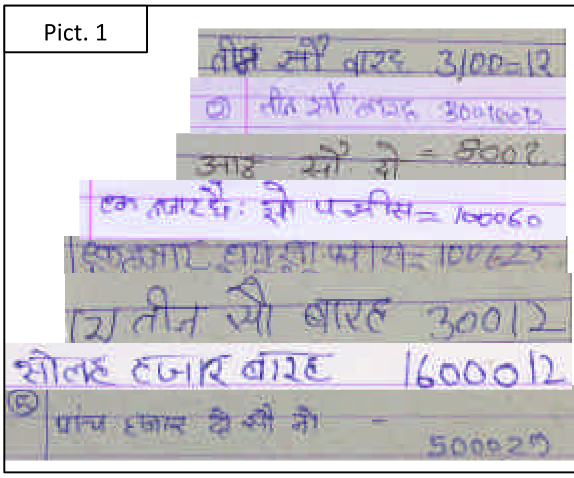
The ‘Ganak’ Exploration: Helping Children to learn about Numbers

By Pramod Maithil, Eklavya

Pramod Maithil, a Research Associate at Eklavya, writes about his study on investigating a tool called Ganak . He proposes that place value work should continue into middle school level with an emphasis on understanding the positional value of digits, and the additive, multiplicative properties of a numbers. He demonstrates his work with kids on sequentially exploring number patterns in various bases through a game based on abacus, pictorially representing them, deciphering their value in the decimal system, and allowing kids to explore their own interpretive way of forming numbers. Pramod can be reached at pramod.maithil@gmail.com

Pict. 1



At times children make sense of mathematical concepts differently which we call “mistakes”. However, these mistakes highlight the children’s understanding of concepts and gaps therein . During one of my interactions with children in 6th grade, I found that they weren’t clear in the standard numerical representation of the numbers. I noticed confusion on using zeros while representing numbers. For e.g., three hundred and twelve was represented as 30012 (Pict. 1). My premise was that their difficulty lies with a basic characteristic of the number system we use now and its written form. This classroom experience led me to go through some literature, documents & materials on mathematics education & particularly on teaching Place Value or Positional Notation System.

With a base 10 (often referred to as decimal system) system we need only ten symbols (1, 2, 3, 4, 5, 6, 7, 8, 9 and 0). All these symbols & all possible combinations of these symbols are associated with unique numbers. Children often fail to differentiate between the face value of these symbols (or digits) and the actual value of the digit.

Historically, the tally system was often used to represent counting. We sometimes still use it. For e.g,

 = 12.

Recognizing this as a lengthy process, people created more

What is place value system?

There are few terms used for this – The decimal number system & the place value system OR the positional notation system. I found a very clear definition written by Sharon R. Ross

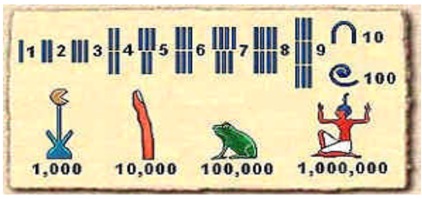
“Our numeration system is characterized by the following four mathematical properties:

1. **Additive property** – The quantity represented by the whole numeral is the sum of the values represented by the individual digits.
2. **Positional property** – The quantities represented by the individual digits are determined by the positions that they hold in the whole numeral.
3. **Base-ten property** – The values of the positions increase in powers of ten from right to left.
4. **Multiplicative property** – The value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position.”

Example 1324 = 10^3x1 + 10^2x3 + 10^1x2 + 10^0x4

www.questia.com, “Place Value: Problem Solving and Written Assessment” by Sharon R. Ross (2002).

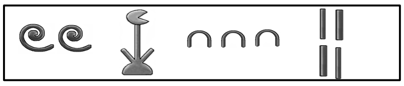
efficient benchmarks such as using new symbols at fixed intervals (5 or 10). Let’s look at the Egyptian system:



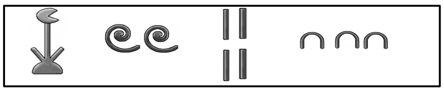
Using these symbols if I need to write one thousand two hundred thirty four ,it will be

If we follow the Egyptian system, we will need to keep generating infinite new symbols for ten, hundred, and thousand so on. Moreover it does not require a specific position for the symbols. In other words the symbols can be placed in any order and still represent the same number.

For e.g.,



Or



All these combinations stand for the number one thousand two hundred thirty four because there is no positional value associated with the individual symbols. Each symbol has the same value irrespective of where it is positioned. The idea of zero was also not needed in this system. Both these systems differ significantly from the decimal system in use today. They do not possess a positional or multiplicative property.

We traditionally teach the concept of place value as per the school textbooks at primary grades. Children are asked to write the numbers in columns marked out as units, tens and hundreds. In the alternative approaches prevalent in India, different concrete materials such as Dienes blocks and matchstick bundles are used. (Usha Menon, Episteme-1)[1]. These manipulatives help children understand grouping and give them a concrete representation of a number. But this still falls short of “connecting” the positional, additive, and multiplicative properties of digits and instead focuses on the mechanics of writing digits in appropriate position. Curriculum in India emphasize that the entire number system is to be taught in the elementary level but I strongly see the need for place value work to continue at middle school level as well.

In Bal Vaigyanik (HSTP’s Class 6 textbook), at the middle school level, the open abacus, *Ganak* is used as a tool to teach decimal & place value. The *Ganak* consists of thin rods vertically set to fit exactly 9 beads each. We begin by placing beads in the right most rod. After 9 beads are set, we take out all the beads and place 1 bead in the second rod from right. This 1 bead at second position represents 1 in the tens place. As the unit place is empty it represent the ‘0’ in the numerical representation of 10.

From my own experience with Bal Vaigyanik, any activity with *Ganak* usually stops at highlighting the number indicated in each rod. I wondered if *Ganak* could be used to connect the positional nature of a number to its additive and multiplicative relation with

---

[1] *The Teaching of Place Value –Cognitive Consideration,s* presented by Usha Menon: www.hbcse.tifr.res.in/episteme1/allabs/ushaabs.pdf

other numbers. Further, I feel that the strength of positional notation system in base ten can be seen only when we expose numbers in different bases, in which we show the gradual increase & shift in the position and the acquisition of value accordingly. Zoltan P. Dienes[2] says:"….*different bases should be used at the start, and to facilitate understanding of what is going on, physical materials embodying the powers of various bases should be made available to children*."

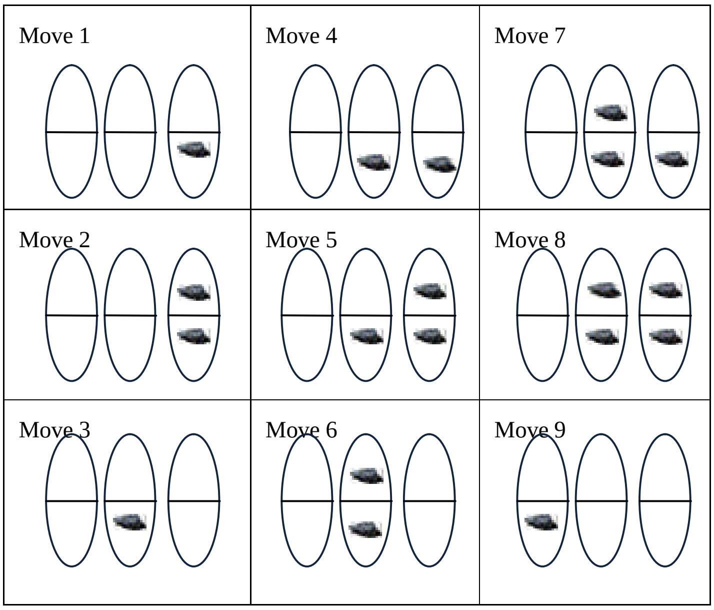


Table 1: *Do khand wala khel*

To start with, I designed a game of base three mimicking the *Ganak.* The game was designed to give exposure to the positional notational system of different bases without explicitly mentioning the use of bases to the students. In this article, I call it a base-three game. I selected a 6th grade classroom in a village middle school with 22 boys & girls[3]. The school usually did not use any manipulatives to teach place value. Before introducing *Ganak,* I spent ~1 month with them using the Number Mala and the Pebble Card as materials for this study. The children built number sense using various counting, addition and subtraction games on the Number Mala and Pebble Card[4]. These games allowed them to develop number sense, grouping, and base-10 property. Taking this as the baseline for their work, we switched to the Ganak game to research sequencing of numbers, positional property of digits, and additive-multiplicative reasoning.

The rest of the article details the interventions with this game as a necessary step before reintroducing base 10 *Ganak* with proper understanding of place value system. Table 1 highlights the base 3 game. The students related to it as a game of two parts - "*Do khand wala khel". *The game involves placing a pebble at each column, starting form the right

---

[2]The Dienes blocks http://www.zoltandienes.com/what_is_a_base.pdf

[3]Throughout the interactive study, there were consistently 10-12 students present in class.

[4] Khushi-Khushi classes 1-5, Primary textbook and workbook for Prathamik Siksha Karyakram (Prashika), developed by Eklavya

most. At the third, sixth & ninth moves, they have to consolidate the moves and replace it by a pebble to the next box or column.

The children played this game several times in base 3 until they recognized the patterns of play/position. In doing so, they were able to associate the representation with a unique number of moves. After several rounds, I concentrated on naming the column (position) as related with the number of moves. They agreed that each column could be numbered based on the least moves it represented. For e.g., the second column for a base 3 game was numbered 3, the third column was numbered 9 and so on (Pict. 2.). The students recognized that the next two columns could be numbered as 27 and 81 respectively.

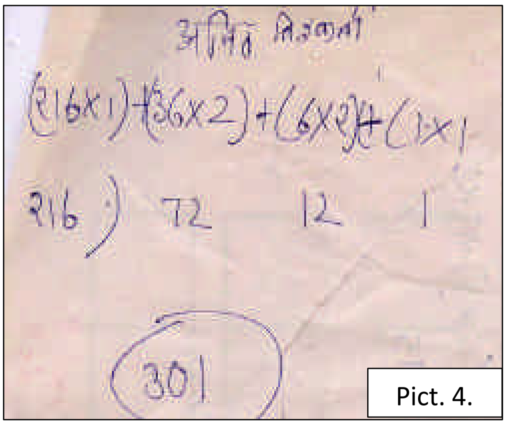
Pict. 2.

I assessed the students by pictorially showing them the representation for 28 moves (in base 3) and asked them to decode the number of moves it represented . Then I gave them a set of blank templates (for base 3) and asked them to fill in the sequence of representations until 34 moves. They used worksheets, chalkboards or just paper to work out these using additive and multiplicative reasoning. Students numbered the sequence of moves using Indo-Arabic digits and recognized the pattern of digits for different bases. For e.g., they recognized that they only need to use the digits 0,1,2 for a base 3 system. This was a fun exploration for them. It was exciting to see children busy exploring new patterns and mathematizing.

Pict. 3.

I further challenged them by increasing the number of parts in each column to 3 (which signifies base 4) and higher bases. They worked in groups & designed their own games by extending the parts in the boxes, and progressed through different bases. They were able to represent moves pictorially, name each column, using the method for the earlier base 3 game and decode the number of moves for various representations using additive and multiplicative reasoning skills(Pict. 3.).

I had realized that children were effortlessly using additive-multiplicative reasoning to find connections between the pictorial representations and the number of moves(decimal value). I wondered if kids could understand equations using the extended notation to relate the different base numbers to the number of moves. I showed them a few simple examples using base 3, such as, $(102)_3$ = (1 x 9) + (0 x 3) + (2 x 1) = 11 moves



Pict. 4.

The students readily caught hold of such expressions and were able to convert other bases back to a number in the decimal system using this method. Thus far, the students were working in small groups and sharing their work. I asked students to work individually to be able to assess their understanding. I repeated the work with sequencing of moves, naming convention, picture representation & then the numerical representational with the 3-base game and recorded their work (Pict. 4).

Towards the end, we returned to base 10 work. My intention was to see if students could recognize the connection between the positional nature of digits and the place value of each column. We extended the game to have 9 parts in each column and then used *Ganak* as a tool. Now the children found it very exciting to realize the direct correlation between the decimal system numbers and the digits in each position. There was no need for any extra work for figuring out the number of moves when interpreting the pictorial representation!

This pilot study was interesting to see how *Ganak* can be utilized for the purpose of building positional value of digits, and additive, multiplicative reasoning skills. I am left with a few issues/questions on a) Why and how should place value should be taught? b)will this kind of intervention help kids understand numbers better in the future? c) what grade should this work be introduced in? d)how do such detailed interventions gel with prescribed syllabus, teacher's time commitments and school schedules? With further collaboration, this study[5] can be treated as an attempt to recommend *Ganak* to educational agencies as a efficient method for developing an understanding of place value.

[5] I would like to thank ICICI Center for Elementary Education, Pune for facilitating this study, & Dr. Anju Saigal (ICEE) for guiding me in this research method. I am also hearty thankful to Ajay Sharma (USA), Anjali Narhonha (Eklavya), Amitabh Mukharji (DU), CN Subramanian (Eklavya), H K Diwan (VBS), Jayasree Subramanian (Eklavya), Kamal Mahendru, K Subramaniam (HBCSE), Maheen(Bhopal), Rakhi (TISS), Reshma Madhusudan (Learning Network), Sushil Joshi (Hoshangabad), TulTul & Rajesh (Eklavya), Vijay Verma (Eklavya) and of-course the children & teachers of middle school, Kulamari, who support me at various stages till this article was written.