Session 6, Part A:

Models for Multiples and Factors

In This Part: The Venn Diagram Model | Finding Prime Factors | The Area Model

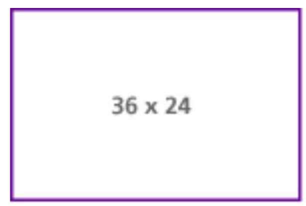
The area model makes the process of finding GCFs and LCMs visual. Note 5

Greatest Common Factor
If we think of the numbers 24 and 36 as the dimensions of a rectangle, then it follows that any common factor could be the dimensions of a square that would tile that entire rectangle.

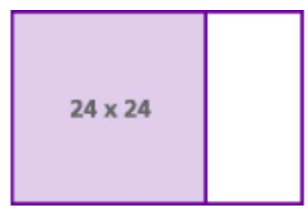
For example, a 1-by-1 square would tile the 24-by-36 rectangle without any gaps or overlaps. So would a 2-by-2 or a 3-by-3 square. Notice that these numbers are all common factors of 24 and 36.

To determine the GCF, we want to find the dimensions of the largest square that could tile the entire rectangle without gaps or overlap. Here's one quick method.

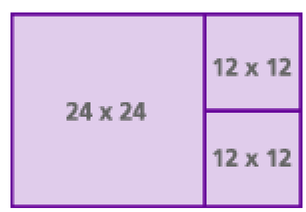
Start with the 36-by-24 rectangle:



The largest square tile that fits inside this rectangle and is flush against one side is 24 by 24. Only one tile of this size will fit:



The largest square tile that fits inside the remaining rectangle and is flush against one side is 12 by 12. Two tiles of this size will fit. The original rectangle is now completely filled:



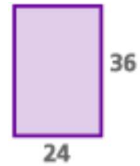
Note that the 24-by-24 square could also be filled with the 12-by-12 tiles, so 12 by 12 is the largest tile that could fill the original 24-by-36 rectangle; therefore, 12 is the GCF of 24 and 36.

Least Common Multiple
Conversely, if we think of 24 and 36 as the dimensions of a rectangle that could tile a square, then it follows that any common multiple could be the dimensions of a square that could be tiled by this rectangle.

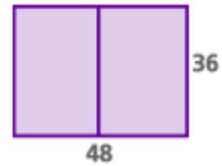
For example, since 24 • 36 = 864, a square that is 864 by 864 could be tiled by the 24-by-36 rectangle. The LCM of 24 and 36 would be the dimensions of the smallest square that could be tiled by the 24-by-36 rectangle.

Here's a quick method for determining the LCM.

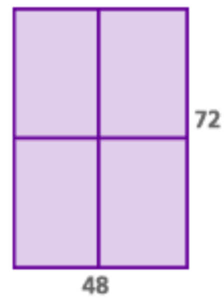
Start with the 24-by-36 rectangle. Your goal is to make a square tiled with rectangles of these dimensions:



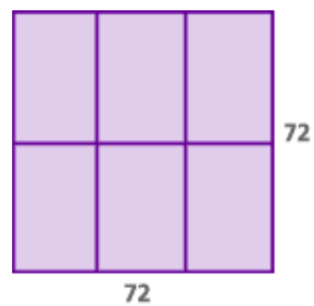
Since the width (24) is less than the height (36), add a column of tiles to the right of the rectangle (in this case, one tile). This makes a 48-by-36 rectangle:



The width (48) is now greater than the height (36), so add a row of tiles under the existing rectangle (in this case, two tiles). This makes a 48-by-72 rectangle:



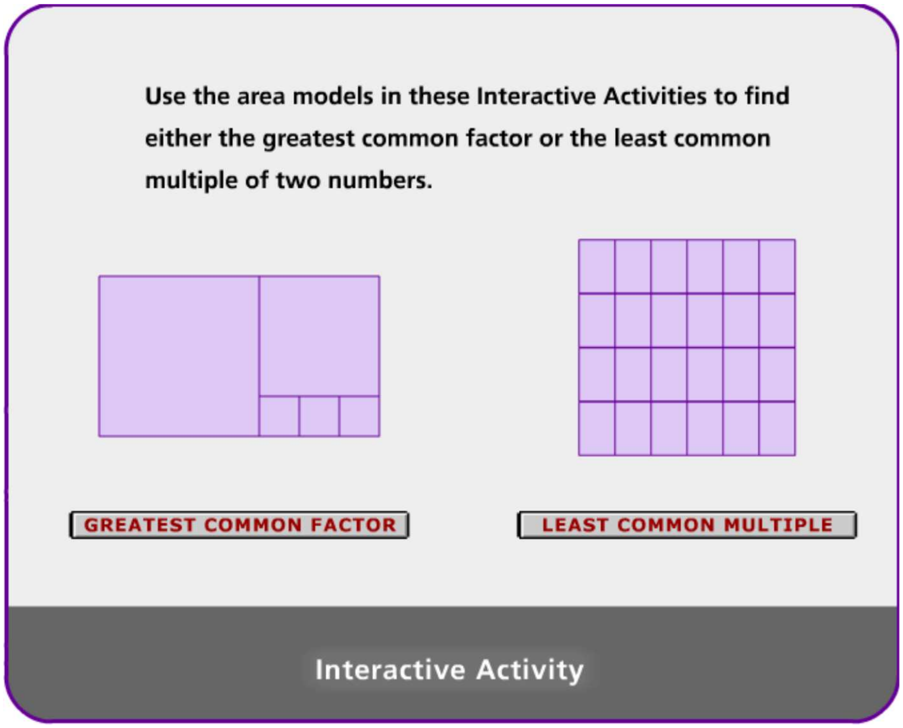
The width (48) is now less than the height (72), so add another column (two tiles) to the right of the existing rectangles. The dimensions are now 72 by 72 -- and you've made a square!



The 72-by-72 square is the smallest square that can be tiled with a 24-by-36 rectangle. Therefore, the LCM of 24 and 36 is 72.

Use the following Interactive Activity to answer Problem A5.

This activity requires the Flash plug-in, which you can download for free from Macromedia's Web site. For a non-interactive version of this activity, use graph paper when drawing the squares and rectangles you wish to represent to ensure that the dimensions of the shapes are precise.



Problem A5

SOLUTION

Use the area model to find the GCF and LCM of the following:

- **a.** 30 and 42
- **b.** 18 and 30

PLAY VIDEO

Video Segment

In this video segment, Ben and Doug use the area model to find the GCF for two numbers, following the analogy of tiling a rectangle with the biggest square they can fit. Watch this segment after you've completed Problem A5.

Notice that the teachers omitted one step and didn't use the square with the dimensions of 12-by-12 to tile the 12-by-30 rectangle. Think about why going through all the steps will ensure that the result will be the largest common factor rather than just any common factor.

If you are using a VCR, you can find this segment on the session video approximately 6 minutes and 4 seconds after the Annenberg Media logo.

Problem A6

WRITE AND REFLECT

Can you explain in your own words why the area model works?

PLAY VIDEO

Video Segment

Here, Ben and Doug use the area model to find the LCM for two numbers, following the analogy of finding the biggest square that can be tiled with a rectangle whose dimensions are the two original numbers. Notice the connection they make between the area model and Venn diagrams.

If you are using a VCR, you can find this segment on the session video approximately 8 minutes and 32 seconds after the Annenberg Media logo.

Next > Part B: Looking for Prime Numbers

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